The Z-12 Performance Advantage

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Biography

Robert Lorenz graduated from the University of California at Berkeley in 1987 with a B.Sc. in Electrical Engineering and Computer Science. Since then, he has worked at Ashtech in the area of signal processing and Integrated Circuit design. He is currently pursuing graduate studies at Stanford University in the field of Electrical Engineering.

Sergei Gourevitch is a Senior Scientist and Manager of Software Development with Ashtech. Dr. Gourevitch was awarded a Ph.D. in experimental particle physics by Case Western Reserve University. After a term of post-doctoral research at Brandeis University, he joined the radio interferometry group at MIT While at MIT, he was part of the group which demonstrated the feasibility of precise GPS geodesy and pioneered the technique. He made important contributions to all areas of GPS geodesy and navigation.

Abstract

The "Z" signal processing technique has proved to be an effective and robust technique for affording the civilian user access to dual frequency GPS data.

All methods of tracking the GPS Y-code signals, without knowledge of the Y-code proper, result in some SNR degradation. This paper describes the "Z" technique, presents a straightforward analytical model for computing this SNR degradation, and shows SNR data in support of these calculations.

Introduction

In this paper, we describe the "Z" signal processing technique. We start out with a mathematical description of the GPS signals and follow them through the receiver, focusing on SNR. We develop the idea that the Z technique results in the same observables as an authorized receiver. We develop a simple but accurate model for this technique. From this model, we see the SNR improvement over cross-correlation is 13 dB. Some data showing SNR versus elevation is presented. To demonstrate the efficacy of this technique, plots of double differenced code and carrier phase data are presented taken at power levels below the minimums specified by the GPS ICD-200B.

GPS Signal Structure

Fundamental Frequency f_0 =10.23 MHz

Nominal L1 Frequency = 154 f_0 = 1575.42 MHz

Nominal L2 Frequency = 120 f_0 = 1227.6 MHz

P Code chip rate = f_0

C/A Code chip rate = $\frac{f_0}{10}$

Encryption Code symbol rate $\approx \frac{f_0}{20}$

Telemetry Data rate = 50 Hz

The signal leaving the GPS satellite can thus be modeled as:

$$s(t) = A \cdot C(t) \cdot D(t) \cdot \sin(2\pi f_{L1} t) + \frac{A}{\sqrt{2}} \cdot P(t) \cdot E(t) \cdot D(t) \cdot \cos(2\pi f_{L1} t) + \frac{A}{2} \cdot P(t) \cdot E(t) \cdot D(t) \cdot \cos(2\pi f_{L2} t)$$

where :

A = Transmitted Signal Amplitude
C(t) = C/A code
D(t) = telemetry data
P(t) = P code
E(t) = encryption data when AS is turned on
1 when AS is turned off

The components of the received signal corresponding to the P-code parts can be modeled as:

$$L_1(t) = \frac{A_r}{\sqrt{2}} M_1(t - \rho_1(t)) \cdot e^{j\Phi_1} + N_1$$

$$L_2(t) = \frac{A_r}{2} M_2(t - \rho_2(t)) \cdot e^{j\Phi_2} + N_2$$

The real part of the exponential corresponds to the in-phase component (the signals are mixed with two quadrature components of the NCO) and the imaginary part corresponds to the quadrature component.

$\Phi_{1(2)}$ is the phase difference (in radians) between the signal being tracked and the phase of the Numerically Controlled Oscillator (NCO). It is the job of the loop filters to keep this difference small.

A_r received C/A code signal amplitude

$M_{1(2)}$ $= P(t) \cdot E(t) \cdot D(t)$. These are the modulations on the two carriers.

$\rho_{1(2)}$ is the group delay, through the propagation medium and the receiver, at L1(2).

$N_{1(2)}$ is the additive noise present at the L1(2) frequencies.

Cross-Correlation

If the receiver delays the L2 signal, with respect to the L1 Signal, by an amount t, and then mixes them, the resultant is:

$$L_2(t+\tau)^* \cdot L_1(t) = \frac{A_r^2}{2\sqrt{2}} M_2(t - \rho_2(t) + \tau) \cdot \{$$
$$M_1(t - \rho_1(t)) \cdot e^{j\Phi_2 - \Phi_1} + N_2^* \cdot N_1 \ \}$$

+ cross terms

where * implies complex conjugation.

Let's examine the above expression closely:

- M1 and M2 are both real and almost identical. All the modulations are multiplications by +1 or -1. The cross-correlation term $M_2 \cdot M_1$ is approximately triangular in the group delay difference $t_g = \rho_2 - \rho_1$,
- maximizing the peak value of the received power measures the group delay difference.
- We have ignored the phase change due to the delay t. As the signals involved are at audio frequency at this point, the change in phase due to a change in delay of many nanoseconds is negligible.
- As the standard deviation of the noise is much larger than the signal, at this point, the cross terms, which are noise, are negligible compared to $N_2^* \cdot N_1$.
- Until the detection, (the cross correlation), the signal paths must support the full 10 MHz bandwidth of these spread spectrum signals. This directly affects the noise spectral density around the difference frequency.

Z-12 Architecture

The block diagram is shown in figure 1. The input $\cos(\phi_{s1(2)} - \phi_{n1(2)})$ and $\sin(\phi_{s1(2)} - \phi_{n1(2)})$ are the real and imaginary components of the phase difference between the L1(2) signal being tracked and corresponding NCO, i.e., $\mathrm{Re}(e^{j\Phi_{1(2)}})$ and $\mathrm{Im}(e^{j\Phi_1(2)})$ respectively. The Edge1 and Edge2 signals are synchronous with the local P-code replicas. They control the start and stop times of the first accumulation. E,P,L code1(2) refer to Early, Punctual, and Late replicas of the L1(2) P-code.

The Z technique mixes with the code and the carrier, exactly like a standard P-code receiver.

$$S_1(t) = \hat{P}_1(t) \cdot \left[\frac{A_r}{\sqrt{2}} \cdot M_1(t - \rho_1(t)) \cdot e^{j\Phi_1} + N_1 \right]$$

$$S_2(t) = \hat{P}_2(t) \cdot \left[\frac{A_r}{2} \cdot M_2(t - \rho_2(t)) \cdot e^{j\Phi_2} + N_2 \right]$$

If we multiply by the (suitably timed) locally generated P-code, the P code "collapses." The signals become:

$$S_1(t) = \frac{A_r}{\sqrt{2}} \cdot R_{P\hat{P}}(\tau) \cdot E(t - \rho_1(t)) \cdot D(t - \rho_1(t)) \cdot e^{j\Phi_1} +$$
$$\hat{P}_1(t) \cdot N_1$$

Similarly for L2, we have:

$$S_2(t) = \frac{A_r}{2} \cdot R_{P\hat{P}}(\tau) \cdot E(t - \rho_2(t)) \cdot D(t - \rho_2(t)) \cdot e^{j\Phi_2} +$$
$$\hat{P}_2(t) \cdot N_2$$

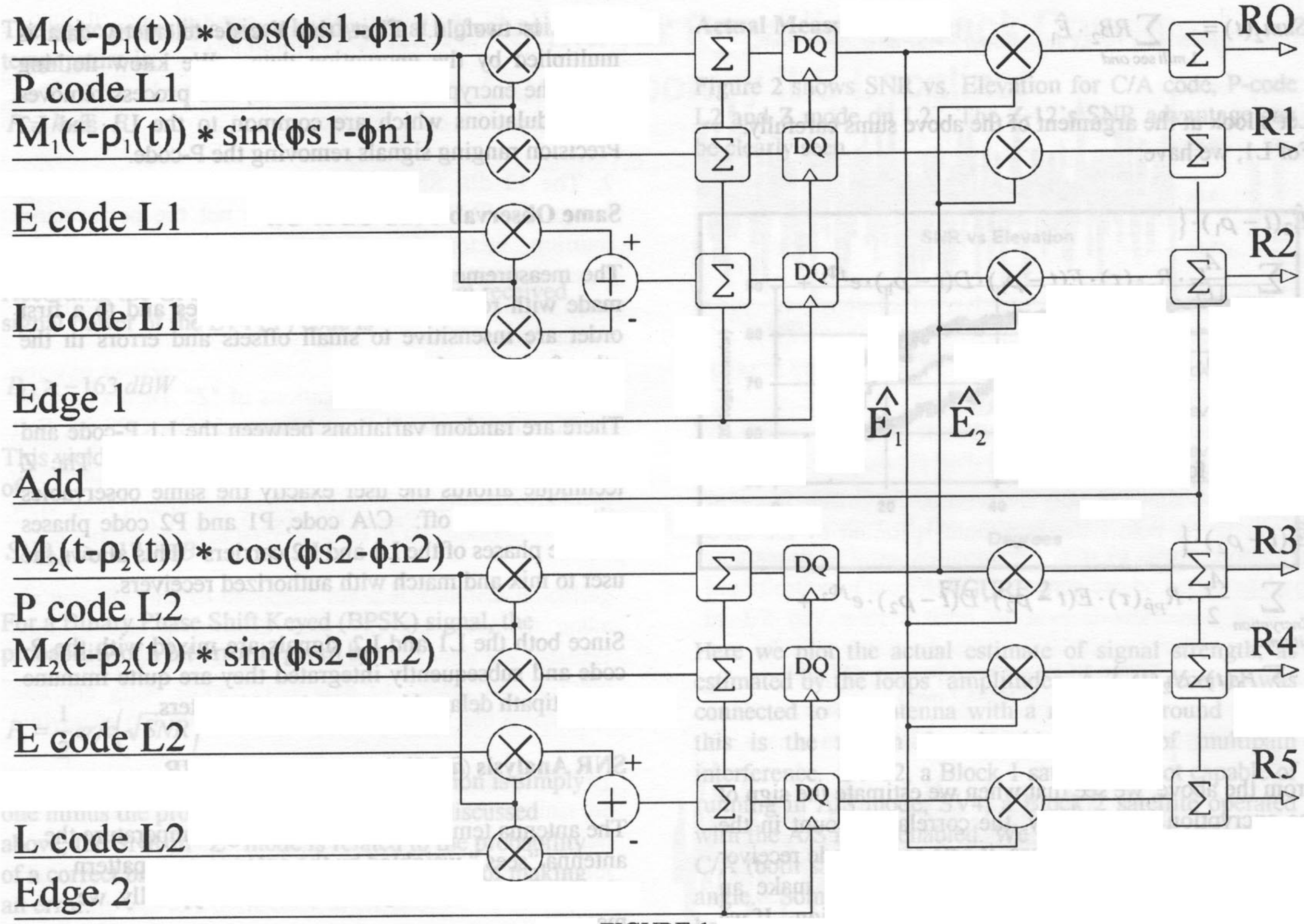

FIGURE 1

where $R_{P\hat{P}}(\tau)$ is the cross correlation function (versus lag) between the incoming P-code signal and the local P-code estimate. As we are able to track the P code on L1 and L2 to within a small fraction of a chip (we can), this is a value near one; it is not identically one at zero lag, due to bandlimiting and sampling losses.

The first accumulation is done over a period approximately equal to that of the encryption bit.

$$RB1 = \sum_{\substack{Encryption \\ bit\ time}} \frac{A_r}{\sqrt{2}} \cdot R_{P\hat{P}}(\tau) \cdot E(t-\rho_1) \cdot D(t-\rho_1) \cdot e^{j\Phi_1} + \sum_{\substack{Encryption \\ bit\ time}} P_1(t) \cdot N_1(t)$$

$$RB2 = \sum_{\substack{Encryption \\ bit\ time}} \frac{A_r}{2} \cdot R_{P\hat{P}}(\tau) \cdot E(t-\rho_2) \cdot D(t-\rho_2) \cdot e^{j\Phi_2} + \sum_{\substack{Encryption \\ bit\ time}} P_2(t) \cdot N_2(t)$$

We estimate the sign of the L1 and L2 encryption bits from the punctual, in-phase L1 and L2 accumulations respectively.

$$\hat{E}_1 = sign(\mathrm{Re}(RB1))$$

Similarly, for L2 we have

$$\hat{E}_2 = sign(\mathrm{Re}(RB2))$$

We multiply the L1 and L2 reduced bandwidth signals (accumulations) by the encryption bit estimate derived from the other frequency, and then sum the results of these multiplications. This summation is done in response to the Add signal.

$$Sum_1(t) = \sum_{1\ milli\sec ond} RB_1 \cdot \hat{E}_2$$

Similarly, for the L2 channel, we have:

$$Sum_2(t) = \sum_{1\ milli\sec ond} RB_2 \cdot \hat{E}_1$$

Let's look at the argument of the above sums carefully. For L1, we have:

$$\hat{E}_2(t-\rho_1)\cdot\{ \sum_{\substack{Encryption\\ bit\ time}} \frac{A_r}{\sqrt{2}} \cdot R_{P\hat{P}}(\tau)\cdot E(t-\rho_1)\cdot D(t-\rho_1)\cdot e^{j\Phi_1} + \sum_{\substack{Encryption\\ bit\ time}} P_1(t)\cdot N_1(t)\ \}$$

Similarly, for L2 we have:

$$\hat{E}_1(t-\rho_2)\cdot\{ \sum_{\substack{Encryption\\ bit\ time}} \frac{A_r}{2} \cdot R_{P\hat{P}}(\tau)\cdot E(t-\rho_2)\cdot D(t-\rho_2)\cdot e^{j\Phi_2} + \sum_{\substack{Encryption\\ bit\ time}} P_2(t)\cdot N_2(t)\ \}$$

From the above, we see that when we estimate the sign of the encryption bit correctly, the correlators count in the correct direction. In this case, it acts as a P-code receiver with the telemetry data removed. When we make an error, the correlators count in the wrong direction. If we could always estimate the encryption bit correctly, we would suffer no SNR degradation.

Let's define $P_{c1(2)}$ as the probability of a correct bit estimate based on the L1 and L2 sums respectively. Similarly, $P_{e1(2)}$ are the probabilities of estimate errors. From the above expressions we see that the L1 SNR degradation is related to:

$$Expected\ Value\left(D_1 \cdot E_1 \cdot \hat{E}_2\right) = P_{c2} - P_{e2}$$

and the L2 SNR degradation is related to

$$Expected\ Value\left(D_2 \cdot E_2 \cdot \hat{E}_1\right) = P_{c1} - P_{e1}$$

Because we must estimate this bit in finite time, i.e., the period of the encryption bit, the estimates are noisy; this is why we suffer an SNR degradation. . Typical values for the probability of a correct estimate vary from say 0.58 to 0.78. We will analyze this probability shortly

Even if we could estimate the encryption bit without error, we cannot recover the Y-code data stream. The reason is twofold. First, the Y-code telemetry data is multiplied by the encryption data. We know nothing about the encryption data. Second, this process removes all modulations which are common to the L1 and L2 Precision ranging signals removing the P-code.

Same Observable A/S On or Off

The measurements of carrier phase and code phase are made with respect to the local references and to a first order are insensitive to small offsets and errors in the other frequency channel.

There are random variations between the L1 P-code and C/A code phases at the 1 nS (0.3m) level. The Z technique affords the user exactly the same observables with A/S on or off: C/A code, P1 and P2 code phases and the phases of the L1 and L2 carriers This allows the user to mix and match with authorized receivers.

Since both the L1 and L2 signals are mixed with the P-code and subsequently integrated they are quite immune to multipath delayed by more than 30 meters.

SNR Analysis @ Minimum Received SNR

The antenna temperature is the average temperature the antenna "sees," weighted by the antenna gain pattern plus losses up to the input of the LNA. Typically, we measure:

$$T_{ant} = 160°\ K$$

The receiver noise temperature is primarily a function of the LNA (Low Noise Amplifier) and sampling losses within the receiver. The Z12 receiver uses an adaptive 2 bit A/D converter which contributes little degradation to the noise performance.

$$T_{rec} = 200°\ K$$

Summing, we get:

$$T_{sys} = T_{ant} + T_{rec} = 360°\,K$$

B is the noise bandwidth associated with the encryption bit estimation filter. This can be computed using the Fourier transform of a rectangular integration window to compute the effective noise bandwidth. The noise power associated with the output of the encryption bit estimation filter is then:

$$B \approx 0.5\ MHz$$

The noise power in a given bandwidth at a given noise temperature is:

$$P = k \cdot T \cdot B$$

where k is Boltzman's constant.

$$P_{noise} = k \cdot T_{sys} \cdot B = -146\ dBW$$

According the GPS ICD-200B, the minimum received signal power of the L1 P(Y) code is

$$P_{L1} \geq -163\ dBW$$

This yields a SNR in the bandwidth of the encryption bit of

$$SNR_{bit} \geq -17.0\ dB$$

For a Binary Phase Shift Keyed (BPSK) signal, the probability of a bit error is given by:

$$P_e = \frac{1}{2} erfc\left(\sqrt{SNR}\right)$$

The probability of making the correct decision is simply one minus the probability of an error. As discussed above, the SNR in "Z" mode is related to the probability of a correct bit decision minus the probability of making an error.

$$P_c - P_e = erf\left(\sqrt{SNR}\right) \approx \frac{2}{\sqrt{\pi}}\sqrt{SNR} \approx 0.16$$

This corresponds to a SNR degradation of 15.9 dB. The resulting Carrier to Noise ratio for the L2 signal is then:

$$\left(\frac{C}{N_0}\right) = -166\ dBW + 203\frac{Hz}{dBW} - 15.9\ dB = 21.1\ dB \cdot Hz$$

The improvement in SNR, relative to cross-correlation can be easily computed. We can use the full P-code bandwidth in the above analysis. This corresponds to a factor of 20 in bandwidth and corresponds to a 13 dB degradation in SNR

There must be enough SNR to provide the loop bandwidth to track the ionospheric changes, but the loops associated with the Y-code signals do not need to accommodate the dynamics introduced by motion of the antenna. The loops are aided by the C/A code carrier; however, we want loop bandwidths as high as possible to make the loop transients settle fast.

Actual Measurements

Figure 2 shows SNR vs. Elevation for C/A code, P-code L2 and Z mode on L2. The Z-12's SNR advantage can be clearly seen.

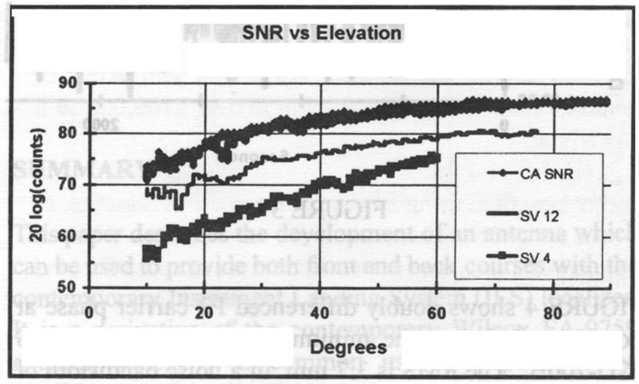

FIGURE 2

Here we plot the actual estimate of signal strength, as estimated by the loops' amplitudes. A Z-12 receiver was connected to an antenna with a minimal ground plane; this is the reason for the high level of multipath interference. SV-12, a Block 1 satellite, is not capable of running in A/S mode; SV4, a Block 2 satellite operated with the A/S mode enabled. We plot the SNR estimate of C/A (both satellites) and of P2 as functions of elevation angle. Some careful measurements, using a simulator, have established that in the units shown along the Y-axis the carrier to Noise density ratio for the C/A code in $dB \cdot Hz$ is given by:

$$\left(\frac{C}{N_0}\right) \approx 20 \cdot \log_{10}(counts) - 30$$

We see that the P2 estimate, for the block 2 satellite, is about 6 dB less than that for C/A. We further see that the maximum SNR degradation, relative to P-code detection of the Block 1 satellite, is less than 15 dB at the lowest elevations, and approximately 5 dB at high elevations.

FIGURE 3 shows doubly differenced P2 code phase at power levels below the minimum specified by the GPS ICD-200B. The RMS is 3 m. in a noise bandwidth of 0.1 Hz.

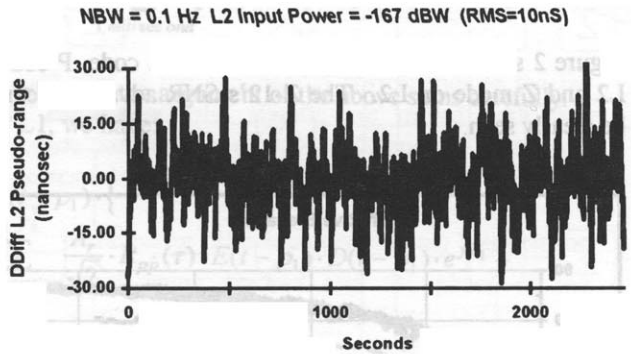

FIGURE 3

FIGURE 4 shows doubly differenced P2 carrier phase at power levels below the minimum specified by the GPS ICD-200B. The RMS is 3.7 mm. in a noise bandwidth of 0.1 Hz

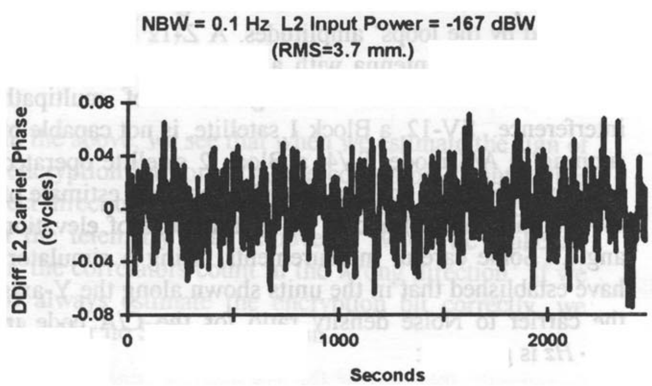

FIGURE 4

Conclusions

In presenting, a detailed description of the method, a theoretical noise analysis and plots of actual measurements, we have demonstrated why we are able to claim that "For the Overwhelming number of users, the Performance of the Z-12, with A/S on, is as good as any other receiver when A/S is off."

The SNR loss, associated with A/S, is typically between 5 and 15 dB. This can be recouped by the increased integration times made possible by C/A carrier aiding.

The cross-correlating method imposes an additional 13 dB SNR degradation. Either the bandwidths must be reduced, making it impossible to track rapidly varying ionosphere, or the observable noise must go up or (usually) both.

The two strengths of the Z-12 are:

1. It tracks exactly the same observables when A/S is on as authorized P-code receivers.

2. The 13 dB. SNR advantage, over cross correlating, brings the SNR up to where it is not the performance limiting factor for geodetic, navigation, or orbit determination purposes.

References

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Lorenz et. al, U.S. Patent Number 5,134,407